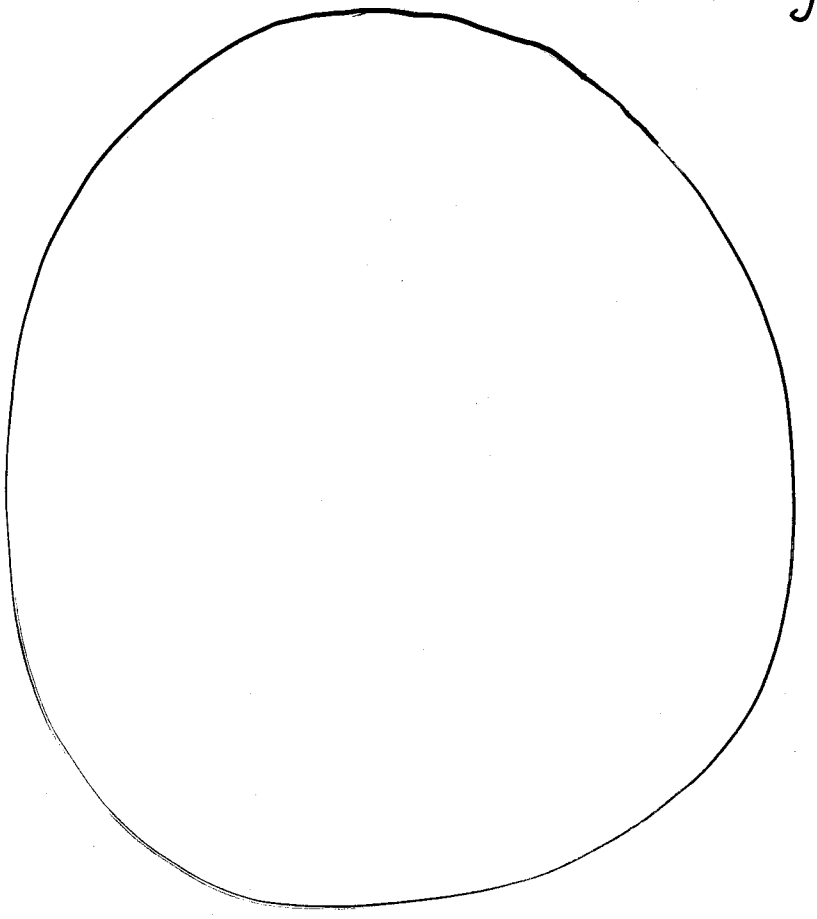
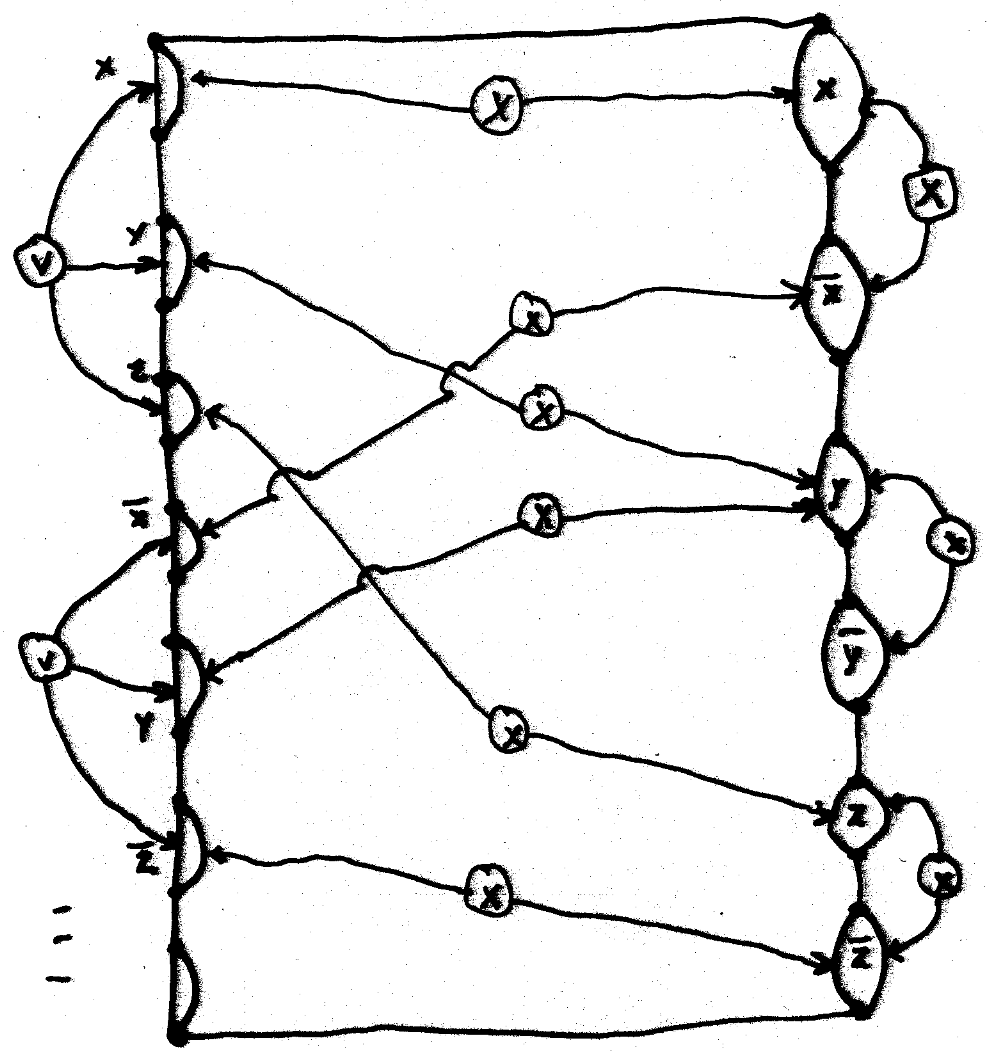
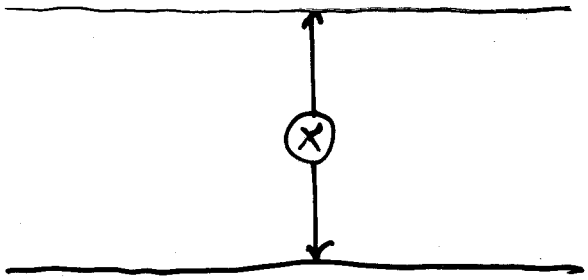
Hamiltonian Cycle: find a simple cycle through all vertices of a graph.

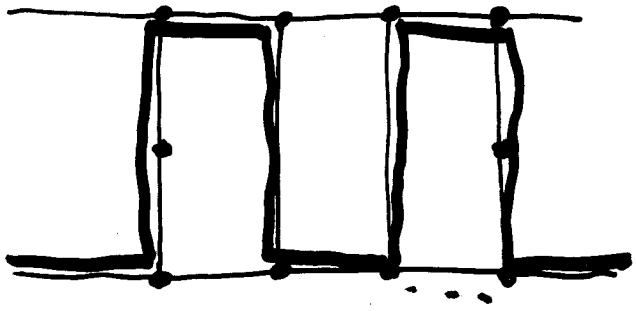


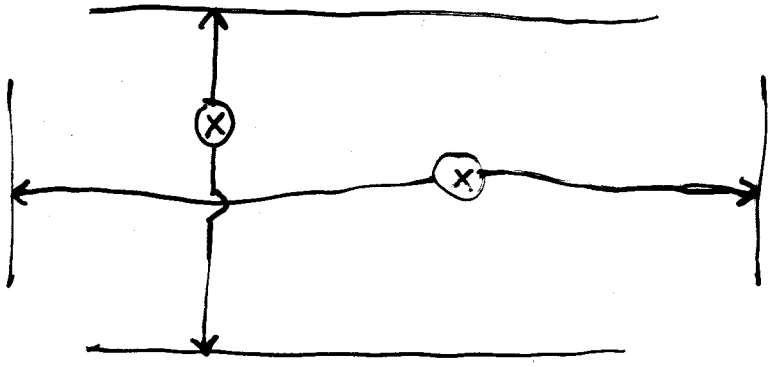
NP-complete: reduction from 3-CNF sat via "gadgets"

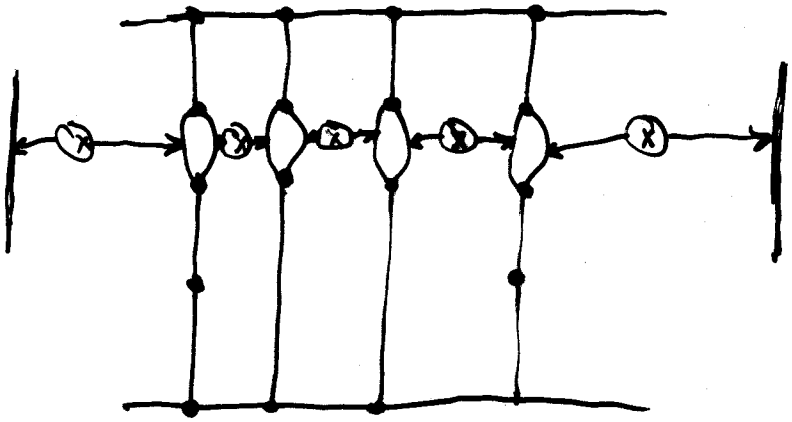
(even for planar graphs, all vertices of degree 3, all faces of 5 or more sides)

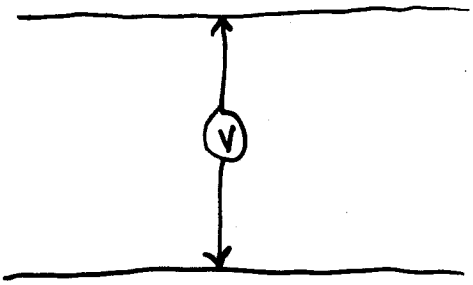


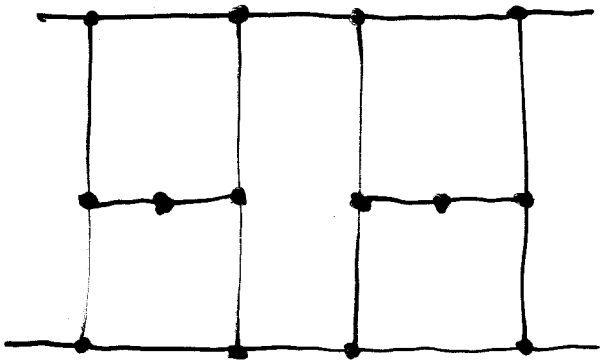


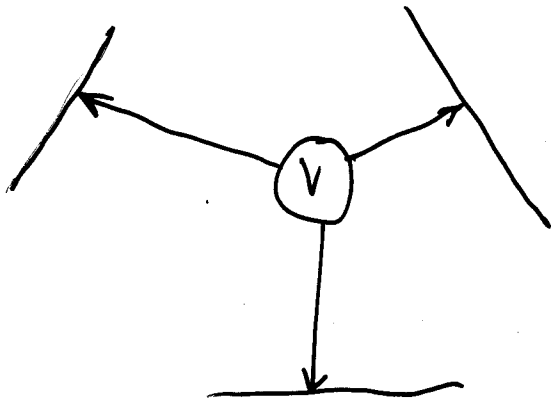


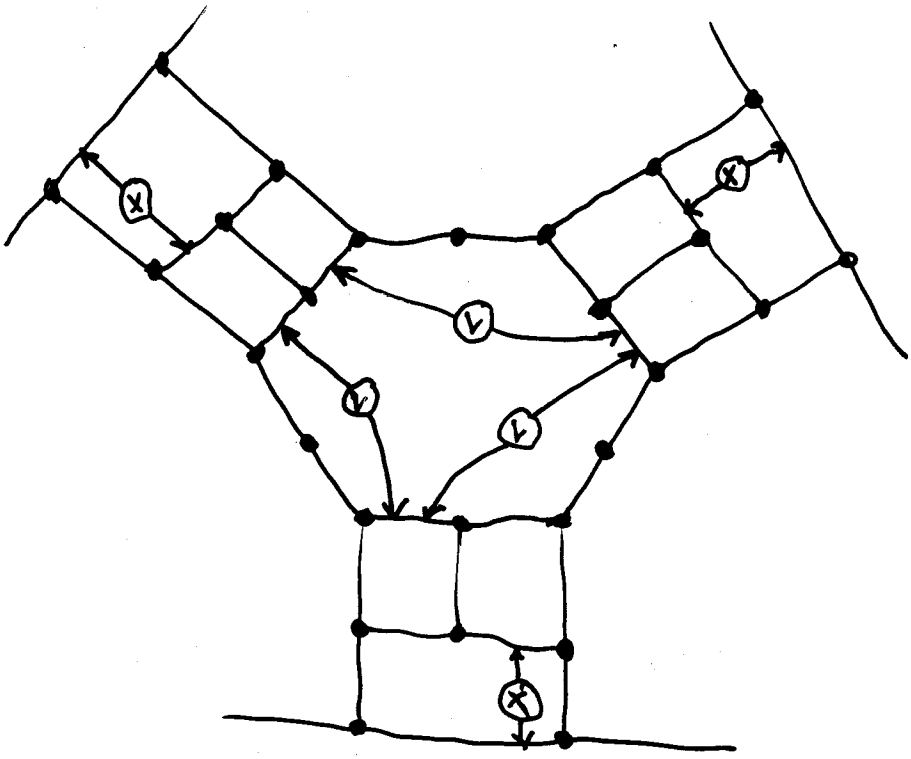


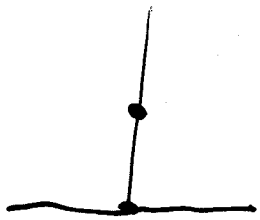


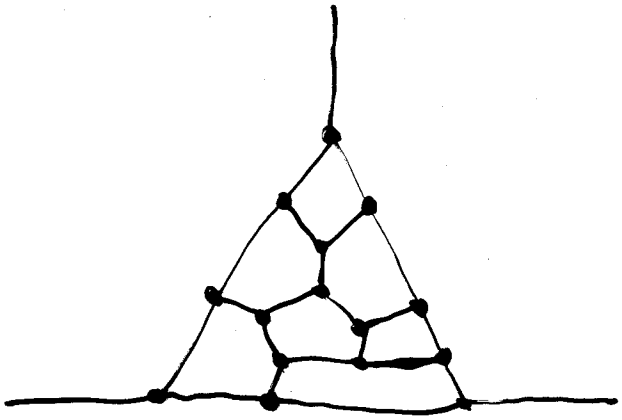

V

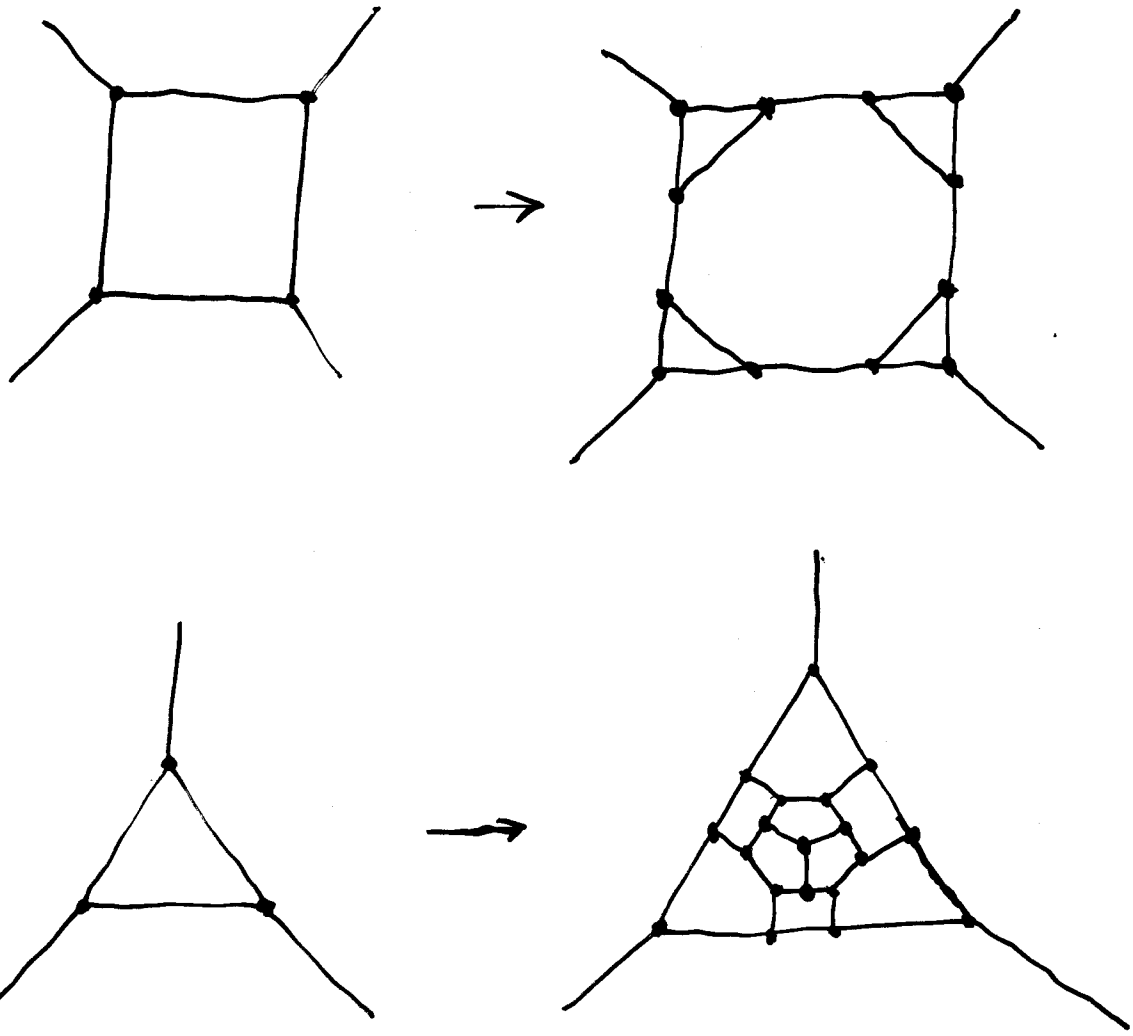



V


X
X
V
V
V
X







Subset Sum is NP-complete

Given n integers, and a target k, is there a subset that sums to exactly k?

$\{2, 5, 6, 8, 9, 12\}$ $\quad k = 31$

yes: 5, 6, 8, 12 $\quad n = \#bits$

(no for $k = 30$)

In NP: subset is proof (verifiable in p-time)

Some NPC problem reducible to subset sum

reduce 3-CNF sat to subset sum

Write numbers base 10

$$(x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (y \vee \bar{z})$$
$$C_1 \qquad\qquad C_2 \qquad\qquad C_3$$

	x	y	z	C_1	C_2	C_3	
x	1	0	0	1	0	0	
$\bar{x}$	1	0	0	0	1	0	
y	0	1	0	0	1	1	← y makes C_2, C_3 true
$\bar{y}$	0	1	0	1	0	0	
z	0	0	1	0	1	0	
$\bar{z}$	0	0	1	1	0	1	← $\bar{z}$ makes C_1, C_2 true
Dummies to get clause columns to sum to 4	0	0	0	1	0	0	
	0	0	0	2	0	0	
	0	0	0	0	1	0	
	0	0	0	0	2	0	
	0	0	0	0	0	1	
	0	0	0	0	0	2	
	1	1	1	4	4	4	= k Required sum

Interpret each row as a base 10 number.

Subset sum has a solution iff formula is satisfiable.